Daily Question – Pure Mathematics - Day 10

Topic: Parametric Equations - Integration

Day 10 Question 1

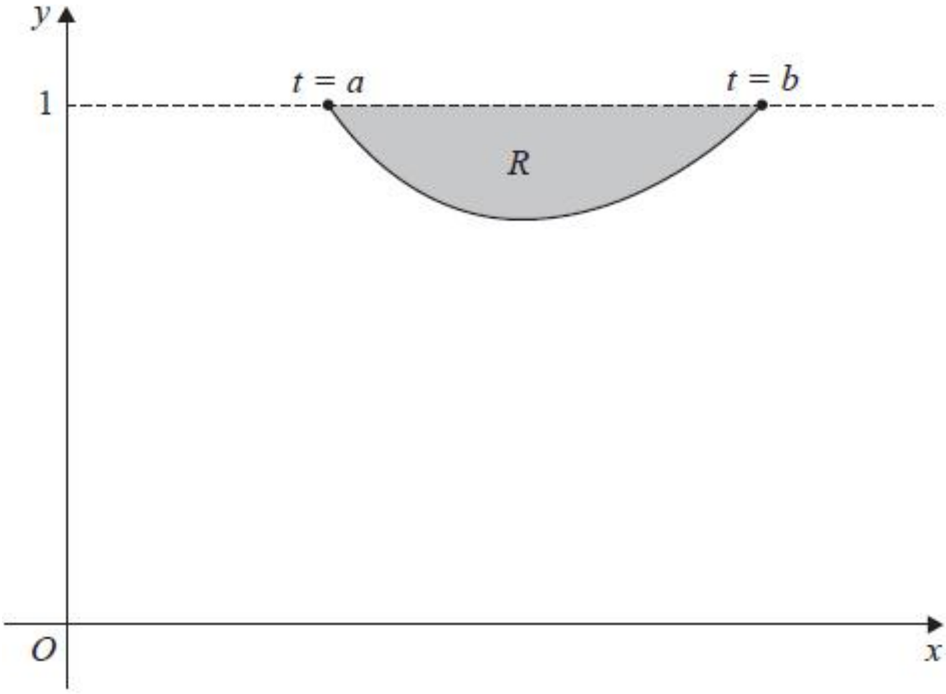

Figure 2

Figure 2 shows a sketch of the curve defined by the parametric equations

$$x = t^2 + 2t \qquad y = \frac{2}{t(3-t)} \qquad a \leqslant t \leqslant b$$

where *a* and *b* are constants.

The ends of the curve lie on the line with equation $y = 1$

(a) Find the value of *a* and the value of *b*

(2)

The region *R*, shown shaded in Figure 2, is bounded by the curve and the line with equation $y = 1$

(b) Show that the area of region *R* is given by

$$M - k\int_a^b \frac{t+1}{t(3-t)}\,\mathrm{d}t$$

where *M* and *k* are constants to be found.

(5)

(c) (i) Write $\dfrac{t+1}{t(3-t)}$ in partial fractions.

(ii) Use algebraic integration to find the exact area of *R*, giving your answer in simplest form.

(6)

(Total for question = 13 marks)

Day 10 Questions 2

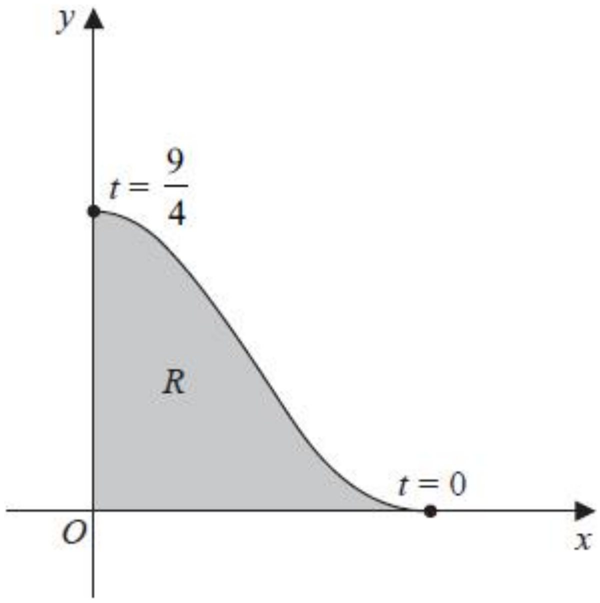

Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = \sqrt{9 - 4t} \qquad y = \frac{t^3}{\sqrt{9 + 4t}} \qquad 0 \leqslant t \leqslant \frac{9}{4}$$

The curve touches the x-axis when $t = 0$ and meets the y-axis when $t = \frac{9}{4}$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the y-axis.

(a) Show that the area of R is given by

$$K \int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81 - 16t^2}} \, dt$$

where K is a constant to be found.

(4)

(b) Using the substitution $u = 81 - 16t^2$, or otherwise, find the exact area of R.

(*Solutions relying on calculator technology are not acceptable*.)

(6)

(Total for question = 10 marks)